

# Holography constrains quantum bounce

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## **Abstract**

Recent work in quantum loop cosmology suggests the universe undergoes a bounce when evolving from a previously collapsing phase. However, with matter sources that obey the strong-energy condition i.e. non-inflationary, the scenario appears to strongly violate the holographic bound on entropy  $S \leq A/4$  during the bouncing phase, where  $A$  now represents the cross-sectional area of the bounce.

We also give a simple argument why any inflationary phase after the bounce is unlikely, contrary to claims in the literature.

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In some recent approaches to quantum gravity the usual singularity in big bang cosmology is possibly replaced by a bounce due to the influence of quantum gravitational effects. As an approximation to this phenomena the standard Friedmann equation for a FRW model e.g.[1,2]

$$H^2 + \frac{k}{a^2} = \rho \quad (1)$$

can become modified such that

$$H^2 + \frac{k}{a^2} = \rho - \frac{\rho^2}{\rho_c} \quad (2)$$

where  $\rho_c$  represents the critical energy scale, typically expected to be around the Planck scale.<sup>1</sup> This behaviour occurs in loop quantum gravity [3-6] although in that case the curvature dependence is actually slightly more involved, but for our purposes this simplified equation will first prove adequate.

A similar description might be obtained with brane models with an extra time dimension [7] although this is probably observationally discounted [8]. Note that a suggested [9] single negative tension brane is not suitable: it differs from eq.(2) by an overall minus sign on the R.H.S. since starting with a positive 5-dimensional Planck mass the negative tension causes the 4-dimensional Newton's constant to become negative cf.[10].

The holography bound [11] on entropy  $S \leq A/4$ , where  $A$  is the corresponding area measured in Planck units appears saturated for black holes but with usual matter there is a stronger restriction  $S \leq A^{3/4}$  - see e.g.[12] for extensive reviews. By choosing a closed radiation dominated FRW universe with corresponding maximum size  $a_{max} \sim A^{1/2}$  this stronger bound is just saturated and  $a_{max}^2 \sim S_r^{4/3}$ , where  $S_r$  represents the corresponding entropy of the radiation present, ignoring some numerical factors that correspond to the number of spin states cf.[13-15].

By starting at the maximum size of a closed radiation dominated universe we then know the amount of entropy initially present, By letting the model collapse we can find the corresponding minimum radius  $a_{min}$  and the corresponding allowed entropy at the bounce  $S_b$ . Since we know the amount of entropy, and assuming adiabatic behaviour so that the entropy remains constant, we can check whether the holography bound  $S \leq A/4$  remains satisfied or not during all the ensuing evolution. Of course, with the standard

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<sup>1</sup> We use Planck units throughout and set numeric factors like  $8\pi/3$  to unity.

Friedmann equation the bound will become violated but this simply represents the impending the big crunch singularity which should now be evaded by the modifications to the equations.

As a first approximation we first keep the usual curvature term  $k/a^2$  on the LHS of eq.(2). Also since the matter behaviour is expected to display its usual form we take  $\rho = \alpha/a^4$ . Then  $a_{max}^2 \sim \alpha$  and assuming  $\rho_c \simeq 1$ , we find  $S_b \simeq a_{min}^2 \sim \alpha^{1/2}$ . Then since  $\alpha^{3/4} > \alpha^{1/2}$  the holography bound is strongly violated i.e.  $S_r > S_b$ . We typically have in mind starting with a large classical universe with  $\alpha \sim 10^{120}$ , so the starting entropy of the radiation is  $10^{90}$  but only a value of  $10^{60}$  should be possible if the holographic bound is still valid across the constricted bounce.

To correctly include curvature actually involves a more complicated equation, but for the case of a massless scalar field the corresponding values are  $a_{max}^2 \sim p_\phi$  and  $a_{min}^2 \sim p_\phi^{2/3}$  [5]. The parameter  $p_\phi$  is here analogous to the previous  $\alpha$ . The corresponding holography bound is now violated, but less severely, since now the corresponding values of entropy are  $p_\phi^{3/4} > p_\phi^{2/3}$ .

A more careful analysis of the relevant equations presented in ref.[5,16] confirms the results and shows that the apparent violation becomes larger as the strong-energy condition boundary is approached and that the bound could become satisfied for ultra-stiff equations of state i.e  $w > 4/3$ , where for a perfect fluid  $p = w\rho$ . Such equations of state have previously been used during the collapsing stage of the Ekpyrotic scenario [17] with certain possible advantages [18].

The use of a closed model was not strictly relevant for this argument since one could start with any finite physical size and derive analogous quantities. The crucial point is that the universe is being constricted into a finite size during the bounce regardless of the underlying geometry  $k$ . This in turn simplifies the application of holography bound compared to various complications that can occur in usual FRW models [12].

The bound might also be satisfied if the critical density  $\rho_c$  is vastly decreased, so the repulsive gravitational effect can intervene sooner, but this is probably anyway inconsistent with other phenomena. There might also be scope to increase the bounce size  $a_{min}$  by employing different quantization procedures that introduce further dependence on the lattice size employed. Although so-called polymer quantization appears to then actually reduce the bounce size [19]. It is worrying, though, that the procedures have already meant to have agreed on the standard Bekenstein-Hawking black hole result  $S = A/4$  when setting the arbitrary Barbero-Immirzi parameter, which in

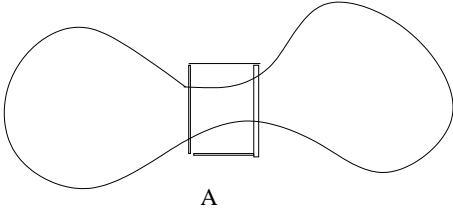


Figure 1: Sketch of the universe “threading the needle”: bouncing at a size below the holographic bound of area  $A$

term gives the corresponding  $\rho_c$  value, see e.g.[20].

If black holes are also present, and during the collapsing phase they might be expected to more easily form and congeal cf.[15], the bound will be much harder to satisfy since the entropy can potentially increase towards  $S \sim a_{max}^2 \sim \alpha \sim 10^{120}$  : essentially the matter during the bounce is being more compressed than in the usual black hole case, such densely entropic states have previously been dubbed monsters [21]. One can see this potential problem by simply relating the mass of the observable universe  $\sim 10^{55} kg$  to the Planck density  $\sim 10^{97} kgm^{-3}$  so the universe could be apparently squashed into a volume  $10^{-42} m^3$  if one could work with Planck density matter: yet the Schwarzschild radius of a supermassive black hole is alone  $\sim 10^{10} m$  so we never remotely deal with Planck energy densities with large black holes: the average density of a large black hole can be small!

Another way to evade this entropy problem would be to have only Planck sized quantities during the collapsing phase i.e.  $\alpha \sim p_\phi \sim 1$  but then the universe, bereft of classical matter before the bounce, is vastly asymmetric cf. [22]. An inflationary phase would have to ensure sufficient entropy production for the later universe.

The question of how much information still remains in the post-bounce phase from the previous collapsing stage will obviously be strongly dependent on preserving entropy across the bounce - in the example with radiation only a fraction  $10^{60}/10^{90} \sim 10^{-30}$  is preserved if the bound is to be satisfied - so only a limited knowledge of the collapsing phase would still be possible cf.[23].

Whether this “eye of the needle” constriction -see Fig.(1)- is too severe and the holography bound should instead be respected; or else other processes intervene to reduce the entropy, will have to be resolved before such a (monster?) quantum bounce can be countenanced.

Let us consider further why the possibility of inflation after the bounce is difficult to envision. Firstly there is an argument using the canonical measure [24] that suggests inflation, driven for example by a massive scalar field  $V(\phi) = 1/2m^2\phi^2$ , is likely if applied at the bounce point [25]. It is known that applying such an argument at Planck energy densities is generally conducive to obtaining inflationary behaviour [26]. But this argument assumes no-prior knowledge is known about the previous state of the system so is akin to the universe suddenly commencing its existence at the bounce itself. The analysis should rather take into account the previous mostly classical evolution of the universe collapsing towards the bounce having started presumably at time  $t = -\infty$ .

Here is where a difficulty arises: if we simply set the field at the minimum of the potential it will typically oscillate <sup>2</sup> about the minimum with a high frequency  $m \sim 10^{-5}$  and as the universe contracts the amplitude of the field  $\phi$  will increase as typically  $\sim a^{-3/2}$  e.g. [1,2,27]. But now whatever the couplings are that typically reheat after any inflationary phase, where again the field oscillates with frequency  $m$ , will interfere and dissipate this high frequency oscillation into particle creation as the universe slowly contracts: so either the couplings are mysteriously absent and the universe would always be dominated by the scalar field and any subsequent inflationary phase could not reheat by the usual oscillating scalar field mechanism; or the reheating effectively occurs during the final stages of collapse preventing any large and *coherent* scalar field being present and no subsequent inflation caused by  $V(\phi)$  would result. This problem has been overlooked cf.[3] because “the reheating has simply been assumed to occur after rolling into the minimum but not when it already was in the minimum” so introducing an asymmetry that is unwarranted. There might be convoluted ways of escaping this dilemma but at first sight it seems there will be entropy production as the universe collapses due to the high frequency oscillations of  $\phi$  together with the presence of any remotely non-zero couplings to other matter. This then suggests the universe will become simply dominated by standard non-inflationary matter as the bounce approaches and so does not allow any inflationary low entropic conditions to naturally occur.

In conclusion, we have outlined problems for the bouncing scenario either

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<sup>2</sup>We ignore the further issue that one might expect the oscillations about the minimum to be out of phase beyond some initial *coherent length scale*, but this would entail using an inhomogeneous metric.

with just standard matter or when a scalar field is included to hopefully drive an extra inflationary phase. Either large initial entropy or growth during the semi- eternal collapsing phase, due to quantum dissipative effects<sup>3</sup>, will tend to cause violation of the holography bound during any possible bounce - so in apparent contradiction with the claims in [28]. Any subsequent useful inflation caused by any generated homogeneous scalar field is not compatible with placing the field simply at the minimum of a potential with, however slight, couplings to standard matter.

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<sup>3</sup>Even if the effects are mainly occurring just seconds before the bounce one generally needs reheating fairly rapidly, so the couplings cannot be excessively weak, to be compatible with nucleosynthesis constraints [1,2].

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